



# Mathematics

## Calculation Progression Policy

**Nursery: 22-36 months**

Selects a small number of objects from a group when asked, for example, 'please give me one', 'please give me two'.

Creates and experiments with symbols and marks representing ideas of number

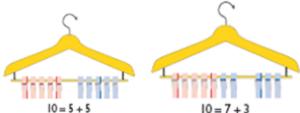
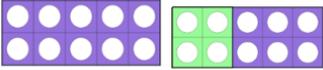
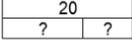
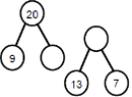
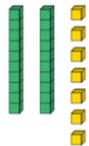
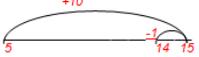
Begins to make comparisons between quantities.

Uses some language of quantities, such as 'more' and 'a lot'

Knows that a group of things changes in quantity when something is added or taken away.

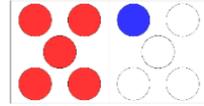
Representations	Key knowledge and vocabulary	Concrete & pictorial Conceptual modelling	Abstract Skills and knowledge	Application across the environment
	<p>Concepts of quantity, equality and inequality.</p> <p>Modelling combining sets of small quantities.</p> <p>Modelling adding to a quantity to make it bigger.</p> <p>Removing objects from a set to show the amount is now smaller.</p>	<p>Natural materials and physical objects in all environments.</p> <p>Pictures to show one or two items.</p> <p>Objects and resources to physically represent a quantity. Images and pictures to represent a small quantity.</p> <p>Using dishes/hoops to make quantities of different values that visually show one set has more than the other. Images of quantities to compare. Which has more?</p>	<p>Spoken number names. <i>One, once, alone, first.</i></p> <p>Mark making and graphics to represent a small number in the context of play.</p> <p>Mark making and graphics to represent a small quantity to compare in the context of play.</p>	<p>Wonderful one and terrific two displays.</p> <p>Hiding objects find one of, or lots of in the sand, across the setting.</p> <p>Matching one item to another then to one image. Repeat with two.</p> <p>Snack time: one piece of fruit to one person, two pieces each</p> <p>Problem solving: "We need one/two each how can we sort the bears?"</p>

# ADDITION KS 1

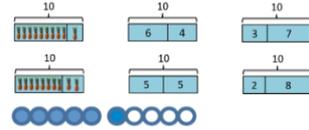
<p>EYFS</p>	<p><b>Reception: ELG 2018</b></p> <p>Numbers to 20: place them in order and say which number is one more or one less than a given number</p> <p><b>Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer</b></p> <p>They solve problems, <b>including doubling</b>, halving and sharing.</p> <p><b>Exceeding:</b></p> <p>Estimation and checking quantities by counting up to 20</p> <p>Combining groups of 2, 5 or 10 or sharing into equal groups</p>					
<p>Year</p>	<p>1</p>	<p>2</p>				
<p>Layers of vocabulary</p>  <p><b>Appendix 1a</b> Beck's Tiers of Vocabulary</p> <p><b>Appendix 1b:</b> Vocabulary book</p>	<p><b>Basic to subject specific (Beck's Tiers):</b></p> <p>+, add, more plus make, sum, total altogether score double, near double one more, two more... ten more how many more to make...? how many more is... than...? how much more is...?</p> <p><b>Instructional vocabulary:</b></p> <p>start from, start with, start at look at point, to show me</p> <p><b>Basic to subject specific (Beck's Tiers):</b></p> <p>+, add, addition, more, plus make, sum, total altogether score double, near double one more, two more... ten more... one hundred more how many more to make...? how many more is... than...? how much more is...?</p> <p><b>Instructional vocabulary:</b></p> <p>tell me, describe, name, pick out, discuss, talk about, explain, explain your method, explain how you got your answer, give an example of... show how you...</p>					
<p>NC 2014</p>	<p>Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs.</p> <p>Using concrete objects and pictorial representations, including those involving numbers, quantities and measures <math>\square</math> applying their increasing knowledge of mental and written methods</p>					
	<p>Concrete, pictorial, abstract</p>					
<p>Developing Conceptual/ Procedural Understanding</p>	<p><b>Number bonds</b></p>  <p><math>10 = 5 + 5</math>      <math>10 = 7 + 3</math></p> <p>We have 10 pegs on the coathangers, how can we split them into 2 groups? Is there another way? How can we be sure we have got them all?</p>   <p>Ten Frames</p>	 <p><math>1 + 1 = 2</math> double 1 is 2      <math>2 - 1 = 1</math> half of 2 is 1</p>  <p><math>2 + 2 = 4</math> double 2 is 4      <math>4 - 2 = 2</math> half of 4 is 2</p>  <p><b>Recognise small quantities</b></p>  <p><b>Count on</b></p>	<p><b>Whole-part model</b></p>   <p>Fill in the missing numbers</p> <p><b>Balance image for concept of equality.</b></p>	<p><b>Base 10</b></p>  <p><b>Whole-part model</b></p> 	<p><b>Adjustment strategy</b></p> <p><math>5 + 9 =</math></p> <p><math>5 + 10 - 1 = 14</math></p>   <p>(Round and adjust)</p> <p><b>Doubles then near doubles</b></p> <p><math>5 + 6 =</math></p> <p><math>5 + 5 + 1 = 11</math></p>	<p><b>Partition and recombine</b></p> <p>Record partitioned steps in number sentences then add mentally.</p> <p><math>40 + 20 = 60</math></p> <p><math>6 + 7 = 13</math></p> <p><math>60 + 13 = 73</math></p> <p>Moving on to:</p> <p><math>46 + 27 = 60 + 13 = 73</math></p>

$2 + \square = 10$        $10 - \square = 3$   
 $5 + \square = 10$        $10 - \square = 9$   
 $\square + 4 = 10$        $10 - 0 = \square$

Hungarian frames



Use the pattern to complete the number sentences.

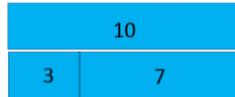


Use bonds of 10 to calculate bonds of 20.



Count on, on number track in 1s.

Develop knowledge of fact families.

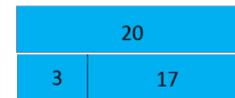


$$10 = 3 + 7$$

$$10 = 7 + 3$$

$$10 - 7 = 3$$

$$10 - 3 = 7$$



$$20 = 3 + 17$$

$$20 = 17 + 3$$

$$20 - 3 = 17$$

$$20 - 17 = 3$$



$$9 = 9$$

$$9 = 8 + 1$$

$$9 = 7 + 2$$

$$8 + 1 = 7 + 2$$

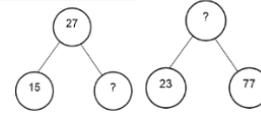


$$10 = 10$$

$$10 = 8 + 2$$

$$10 = 6 + 4$$

$$8 + 2 = 6 + 4$$



Fill in the missing numbers

All answers to be recorded in a number sentence following any informal recording.

Adding more than two numbers

Strategy to include looking for facts or bonds that are useful e.g. bonds up to and including 10, doubles or adding 10 to a given number.

$$6 + 3 + 4 = 13$$

$$6 + 3 + 4 + 7 + 2 = 22$$

Record thinking.

$$7 + 8 =$$

$$8 + 8 - 1 = 15$$

$$47 + 50 =$$

Re-arranging

$$18 + 4 =$$

Tell me what you know about 4, e.g.

$$3 + 1, 2 + 2$$

18 + 4 = Rearrange the 4 into 2 + 2

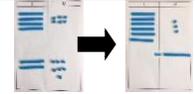
$$18 + 2 + 2 = 20 + 2 = 22$$

59 + 24 = Partition the 24 into 20 + 4 and rearrange the 4 into 1 + 3.

$$\text{So } 59 + 24 =$$

$$59 + 20 + 1 + 3 =$$

$$59 + 1 + 20 + 3 = 83$$



Regrouping the 10.

Balance in the equation

$$14 = 8 + 6, 7 + 6 = 8 + 5$$

$$\square = 13 + 9$$

$$3 + \square + 6 = 16$$

$$14 + \diamond = 15 + 27$$

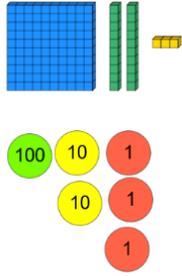
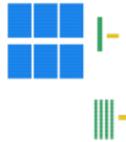
Decision making

Using statements such as:  
Ben did  $14 + 9 = 23$   
How could he have done it?

Known facts	Represent & use number bonds and related subtraction facts within 20 Add and subtract 1 digit and 2 digit numbers to 20, including zero		Recall and use addition and subtraction facts to 20 fluently and derive and use related facts up to 100.	
Essential Knowledge	1 more	Number bonds: 5 and 6	10 more	Number bonds: 20, 12 and 13
	Largest number first.	Number bonds: 7 and 8	Add 1 digit to 2 digit by bridging	Number bonds: 14 and 15
	Add 10.	Number bonds: 9 and 10	Partition second number and add tens then ones.	Number bonds: 16 and 17
	Ten plus ones.	Use number bonds of 10 to derive bonds of 11	Add 10 and multiples of 10.	Number bonds: 18 and 19
	Doubles up to 10.		Doubles up to 20 and multiples of 5.	Partition and recombine.
		Add near multiples of 10.		

# ADDITION KS 2

KS1	<p>Pupils should practise addition to 20 and within to become increasingly fluent. They should use the facts they know to derive others, e.g using <math>7 + 3 = 10</math> to find <math>17 + 3 = 20</math>, <math>70 + 30 = 100</math></p> <p>They should use concrete objects and practical apparatus, such as bead strings and number lines to explore additions including missing numbers. Use pictorial representations such as bar models and whole part diagrams to show additive relationships. 100 squares could be used to explore patterns in calculations such as <math>74 + 11</math>, <math>77 + 9</math> encouraging children to think about ‘What do you notice?’ where partitioning or adjusting is used.</p> <p>Pupils should learn to check their calculations, by using the inverse.</p> <p>They should continue to see addition as both combining groups and counting on.</p> <p>They should use Dienes to model partitioning into tens and ones* and learn to rearrange numbers in different ways e.g. <math>23 = 20 + 3 = 10 + 13</math>.</p> <p>Show understanding that adding zero leaves a number unchanged.</p>								
Year	3			4					
<p>Layers of vocabulary</p>  <p><b>Appendix 1a</b> Beck's Tiers of Vocabulary</p> <p><b>Appendix 1b:</b> Vocabulary book</p>	<p><b>Basic to subject specific (Beck's Tiers):</b> +, add, addition, more, plus make, sum, total altogether score double, near double one more, two more... ten more... one hundred more how many more to make...? how many more is... than...? how much more is...?</p> <p><b>Instructional vocabulary:</b> explain your method explain how you got your answer give an example of... show how you... show your working</p>			<p><b>Basic to subject specific (Beck's Tiers):</b> add, addition, more, plus, increase sum, total, altogether score double, near double how many more to make...?</p> <p><b>Instructional vocabulary:</b> calculate, work out, solve investigate, question answer check</p>					
NC 2014	Add and subtract numbers with up to 3 digits, using formal written methods of columnar addition and subtraction.			Add and subtract numbers with up to 4 digits using the formal written method of columnar addition and subtraction where appropriate. Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.					
Developing Conceptual/ Procedural Understanding	<p><b>Near doubles</b> <math>13+14 =</math> Double <math>13 = 26</math> <math>26+1 = 27</math> or Double <math>14 = 28</math> <math>28-1 = 27</math></p> <p><b>Using known facts</b> <math>40 + 80 = 120</math> using <math>4 + 8 = 12</math> So <math>400 + 800 = 1200</math></p> <p><b>Remodelling strategy</b> <math>243 + 198</math> <math>241 + 200 = 441</math></p>	<p><b>Start with least significant digit</b> 67 <u>+24</u> 11 (7+4) <u>+80</u> (60+20) <u>91</u></p> <p>“7 add 4 equals 11 and 60 add 20 equals 80. <math>1 + 0 = 1</math> and 1 ten + 8 tens = 9 tens”</p>	<p><b>Columnar addition</b></p> $\begin{array}{r} 625 \\ + 48 \\ \hline 673 \\ \phantom{0}1 \end{array}$ <p><b>Teach the carried digit.</b></p> <p><b>Representing problems</b></p>	<p><b>Using known facts</b> <math>40 + 80 = 120</math> using <math>4 + 8 = 12</math> So <math>400 + 800 = 1200</math> and <math>4000+8000=12,000</math></p> <p><b>Remodelling strategy</b> <math>3548 + 1998</math> <math>3546 + 2000 = 5546</math></p> <p><b>Place value materials to represent calculations</b></p>	<p><b>Columnar addition</b></p> $\begin{array}{r} 587 \\ + 475 \\ \hline 1062 \\ \phantom{0}11 \end{array}$ <p>“7 add 5 equals 12. That’s 2 units and 1 ten to carry over. 80 add 70 equals 150 and the 1 ten to carry makes 160. That’s 6 tens and 100 to carry over. 500 add 400 equals 900 and the 1 hundred to carry makes 1000”</p> <p>7648 <u>+1486</u></p>	<p><b>Columnar addition (decimals) in contexts such as money and measurement</b></p> $\begin{array}{r} 12.45 \\ 7.36 \\ + 24.50 \\ \hline 44.31 \\ \phantom{0}111 \end{array}$ <p><b>Representing problems</b> There are 259 more boys than girls in Lucy’s school. If there are 789 girls, how many pupils are there altogether?</p> <div style="border: 1px solid black; width: 100px; height: 20px; margin: 10px auto; text-align: center;">?</div>			
<table border="1" style="float: right;"> <tr> <td style="padding: 2px 10px;">759</td> <td style="padding: 2px 10px;">759</td> <td style="padding: 2px 10px;">+ 259</td> </tr> </table>							759	759	+ 259
759	759	+ 259							

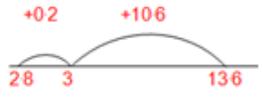
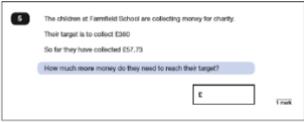
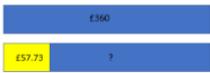
	<p><b>Place value materials to represent 3 digit numbers</b> Base 10 and then place value counters.</p> 	 $\begin{array}{r} 625 \\ + 48 \\ \hline 13 \text{ (5+8)} \\ 60 \text{ (20 + 40)} \\ +600 \text{ (600 + 0)} \\ \hline 673 \end{array}$ <p>All language in the context of the place value and the mental addition of the totals to be done in any order.</p>	<p>There are 334 children at Springfield School and 75 at Oak Nursery. How many children are there altogether?</p>		$\begin{array}{r} 14 \text{ (8+6)} \\ 120 \text{ (40+80)} \\ 1000 \text{ (600+400)} \\ + 8000 \text{ (7000+1000)} \\ \hline 9134 \end{array}$ $\begin{array}{r} 7648 \\ + 1486 \\ \hline 9134 \\ 111 \end{array}$	
Known facts	Derive and use addition and subtraction facts to 100, e.g. $33+ 67 =100$ .		Derive and use addition and subtraction facts (for multiples of 10) to 1000, e.g. $330+ 670=1000$ .			
Essential knowledge	Add single digit bridging through boundaries	Add multiples of 10,100	Fluency of 2 digit + 2 digit	Add multiples of 10, 100 and 1000		
	Partition second number to add	Pairs of 100 (complements of 100)	Partition second number to add	Decimal pairs of 10 and 1		
	Use near doubles to add	Add near multiples of 10 and 100 by rounding and adjusting	Use near doubles to add	Adjust both numbers before adding		
	Partition and recombine		Add near multiples	Partition and recombine		

# SUBTRACTION KS 1

<p>EYFS</p>	<p><b>Reception: ELG 2018</b>            Numbers to 20: place them in order and say which number is one more or one less than a given number            Using quantities and objects, they add and <b>subtract two single-digit numbers and count on or back to find the answer</b>            They solve problems, including doubling, halving and sharing.</p> <p><b>Exceeding:</b>            Estimation and checking quantities by counting up to 20            Combining groups of 2, 5 or 10 or sharing into equal groups</p>	
<p>Year</p>	<p>1</p>	<p>2</p>
<p>Layers of vocabulary</p>  <p><b>Appendix 1a</b>            Beck's Tiers            of            Vocabulary  <b>Appendix 1b:</b>            Vocabulary            book</p>	<p><b>Basic to subject specific (Beck's Tiers):</b>            take away, distance between, difference between, less than. How many more?            How much greater?            How many fewer?            how much more is...? – subtract, take (away), minus, leave, how many are left/left over? how many have gone? one less, two less, ten less... how many fewer is... than...? how much less is...? difference between half, halve = equals, sign, is the same as</p> <p><b>Instructional vocabulary:</b>            start from, start with, start at            look at point, to show me</p>	<p><b>Basic to subject specific (Beck's Tiers):</b>            subtract, subtraction, take (away), minus leave, how many are left/left over? one less, two less... ten less... one hundred less how many fewer is... than...? how much less is...? difference between half, halve = equals, sign, is the same as tens boundary            difference,            partition,            rearrange,            inverse, place value</p> <p><b>Instructional vocabulary:</b>            tell me, describe, name, pick out, discuss, talk about, explain, explain your method, explain how you got your answer, give an example of... show how you...</p>
<p>NC 2014</p>	<p>Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs.</p>	<p>Using concrete objects and pictorial representations, including those involving numbers, quantities and measures ☐ applying their increasing knowledge of mental and written methods</p>
	<p>Concrete, pictorial, abstract</p>	<p>Concrete, pictorial, abstract</p>

<p>Developing Conceptual/ Procedural Understanding</p>	<p><b>Number bonds</b></p> <p>Ten Frames</p> <p>Difference between 7 and 10.</p> <p>2 + <input type="text"/> = 10    10 - <input type="text"/> = 3  5 + <input type="text"/> = 10    10 - <input type="text"/> = 9  <input type="text"/> + 4 = 10    10 - 0 = <input type="text"/></p> <p>Use the pattern to complete the number sentences.</p> <p>6 less than 10 is 4.  <b>Count out, then count how many are left.</b>  Remove from the set.  7 - 4 = 3</p>	<p><b>Count back on a number track.</b>  15 - 6 = 9</p> <p><b>Difference between.</b></p> <p>13 - 8 = <input type="text"/>  8 + <input type="text"/> = 13</p> <p><b>Subtraction-take away</b></p> <p>Jenny's cakes</p> <p>8-3=?  <b>Subtraction-finding the difference</b></p> <p>Peter  Jenny </p> <p>How many more cakes does Peter have than Jenny? 8-3=?</p>	<p><b>Develop knowledge of fact families.</b></p> <p>7=5+2    2+5=7  7-2=5    7-5=2</p> <p><b>Whole-part model</b></p> <p>Fill in the missing numbers</p>	<p><b>Whole-part model</b></p> <p>Fill in the missing numbers  All answers to be recorded in a number sentence following any informal recording.</p> <p><b>Adjustment strategy</b></p> <p>77 - 9 =  77 - 10 + 1 = 67 + 1 = 68</p> <p><b>(Round and adjust)</b>  What is the nearest 10?  55 - 27 =  55 - 30 + 3 = 25 + 3 = 28  91 - 48 =  91 - 50 + 2 = 41 + 2 = 43</p>	<p><b>Re-arranging</b></p> <p>35 - 8 =  Tell me what you know about 8, e.g. 2 + 6, 5 + 3  35 - 8 =  Rearrange the 8 into 5 + 3  So 35 - 5 - 3 = 30 - 3 = 27</p> <p>55 - 27 =  Partition the 27 into 20 + 7 and rearrange the 7 into 5 + 2.  So 55 - 27 = 55 - 20 - 5 - 2 = 35 - 5 - 2 = 28</p> <p><b>Taking away and exchanging</b></p> <p>73 - 46 =</p> <p>What do we know? 76?  Exchange to make about '60 and 13'.</p> <p>Now take away the 46.  73 - 46 = 27</p>	<p><b>Subtract mentally pairs of multiples of 10 using known facts</b></p> <p>60 - 20 = 40 because 6 - 2 = 4</p> <p><b>Partitioning of the second number strategy</b></p> <p>74 - 47  74 - 40 = 34  34 - 4 - 3 = 27</p> <p><b>Balance in the equation</b></p> <p>35 - <input type="text"/> = 31  <input type="text"/> - 12 = 34  20 - <input type="text"/> = 14 - 3  (Open-ended)  18 - <input type="text"/> = 15 - <input type="text"/></p> <p><b>Decision making</b></p> <p>27 - <input type="text"/> = 12  Sam works out 27 - 15 = 12.  How could he have done this?</p>
<p>Known facts</p>	<p>Represent &amp; use number bonds and related subtraction facts within 20  Add and subtract 1 digit and 2 digit numbers to 20, including zero</p>		<p>Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100.</p>			
<p>Essential knowledge</p>	<p>1 less</p>	<p>Number bonds: subtraction 5 and 6</p>	<p>10 less</p>	<p>Number bonds: subtraction 20,12 and 13</p>		
<p>Count back</p>	<p>Number bonds: subtraction 7 and 8</p>	<p>Subtract 1 digit from 2 digit by bridging</p>	<p>Number bonds: subtraction 14 and 15</p>	<p>Number bonds: subtraction 16 and 17</p>		
<p>Subtract 10.</p>	<p>Number bonds: subtraction 9 and 10</p>	<p>Partition second number and count back in tens then ones.</p>	<p>Number bonds: subtraction 18 and 19</p>	<p>Number bonds: subtraction 18 and 19</p>		
<p>Teens subtract 10</p>	<p>Difference between</p>	<p>Subtract 10 and multiples of 10.</p>	<p>Subtract near multiples of 10.</p>	<p>Difference between</p>		
<p></p>	<p></p>	<p></p>	<p>Add near multiples of 10.</p>	<p></p>		
<p></p>	<p></p>	<p></p>	<p></p>	<p></p>		

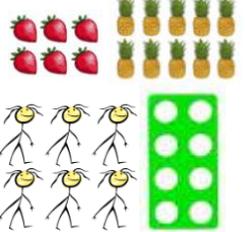
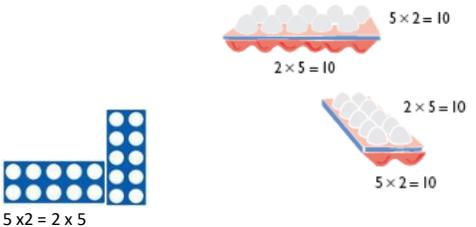
# SUBTRACTION KS 2

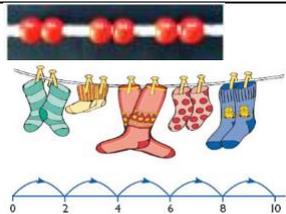
<p><b>KS1</b></p>	<p>Pupils should practise subtraction to 20 and within to become increasingly fluent. They should use the facts they know to derive others, e.g using <math>10 - 7 = 3</math> and <math>7 = 10 - 3</math> to calculate <math>100 - 70 = 30</math> and <math>70 = 100 - 30</math>. Know the effect of zero.</p> <p>As well as number lines, 100 squares could be used to model calculations such as <math>74 - 11</math>, <math>77 - 9</math> or <math>36 - 14</math>, where partitioning or adjusting are used. Pupils should learn to check their calculations, including by adding to check. They should continue to see subtraction as both take away and finding the difference and should find a small difference by counting up. They should use Dienes to model partitioning into tens and ones* and learn to partition numbers in different ways e.g. <math>23 = 20 + 3 = 10 + 13</math>.</p>					
<p>Year</p>	<p>3</p>		<p>4</p>			
<p>Layers of vocabulary</p>  <p><b>Appendix 1a</b> Beck's Tiers of Vocabulary <b>Appendix 1b:</b> Vocabulary book</p>	<p><b>Basic to subject specific (Beck's Tiers):</b> subtract, subtraction, take (away), minus leave, how many are left/left over? one less, two less... ten less... one hundred less how many fewer is... than...? how much less is...? difference between half, halve = equals, sign, is the same as tens boundary, hundreds boundary exchange, carried digits</p> <p><b>Instructional vocabulary:</b> explain your method explain how you got your answer give an example of... show how you... show your working</p>		<p><b>Basic to subject specific (Beck's Tiers):</b> subtract, subtraction, take (away), minus, decrease leave, how many are left/left over? difference between half, halve how many more/fewer is... than...? how much more/less is...? equals, sign, is the same as tens boundary, hundreds boundary, inverse exchange, carried digits</p> <p><b>Instructional vocabulary:</b> calculate, work out, solve investigate, question answer check</p>			
<p>NC 2014</p>	<p>Add and subtract numbers with up to 3 digits, using formal written methods of columnar addition and subtraction. Least significant digit is always dealt with first to establish if the exchange is needed.</p>		<p>Add and subtract numbers with up to 4 digits using the formal written method of columnar addition and subtraction where appropriate. Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.</p>			
<p>Developing Conceptual/ Procedural Understanding</p>	<p><b>Subtract mentally pairs of multiples of 100 using known facts</b> <math>600 - 200 = 400</math> because <math>6 - 2 = 4</math></p> <p><b>Remodelling strategy (keeping the difference the same)</b> <math>502 - 198</math> <math>504 - 200 = 304</math></p> <p><b>Re-arranging</b> Use of apparatus to understand rearrangements, e.g. 55 as 40 and 15(not as part of calculations).</p>	<p><b>Start with least significant digit -decomposition</b></p> $\begin{array}{r} 81 = 80 \ 1 \\ - 57 \ 50 \ 7 \\ \hline \end{array}$ $\begin{array}{r} 81 = 70 \ 11 \\ - 57 \ 50 \ 7 \\ \hline 24 \ 20 \ 4 \end{array}$ <p>"1 subtract 7 is tricky so I will rearrange 81 into 70 and 11. 11 subtract 7 equals 4 and 70 subtract 50 equals 20. 20 and 4 make 24."</p> $\begin{array}{r} 754 \ 700 \ 50 \ 4 \\ - 86 \ \ \ \ 80 \ 6 \\ \hline \end{array}$	<p><b>Columnar subtraction</b></p> $\begin{array}{r} \phantom{0}6 \phantom{0}14 \phantom{0}1 \\ \phantom{0}7 \phantom{0}8 \phantom{0}4 \\ - \phantom{0}2 \phantom{0}8 \phantom{0}6 \\ \hline \phantom{0}4 \phantom{0}6 \phantom{0}8 \end{array}$ <p><b>Emphasis on language of place value, i.e. 14 units subtract 6 units, 14 tens subtract 8 tens, and 6 hundreds subtract 2 hundreds.</b></p> <p><b>Representing problems</b> There are 386 pupils at Oak Primary. If 79 pupils have sandwiches, how many have dinners?</p>	<p><b>Subtract mentally pairs of multiples of 1000 using known facts</b> <math>6000 - 2000 = 4000</math> because <math>6 - 2 = 4</math></p> <p><b>Remodelling strategy (keeping the difference the same)</b> <math>3548 - 1998</math> <math>3550 - 2000 = 1550</math></p> <p><b>Find the difference strategy</b> <math>136 - 28 =</math></p>  <p><math>28 \ 3 \ 136</math></p> <p><math>136 - 28 = 108</math></p>	<p><b>Columnar subtraction</b> <math>2344 - 187</math></p> $\begin{array}{r} \phantom{0}2 \phantom{0}3 \phantom{0}4 \phantom{0}1 \\ \phantom{0}2 \phantom{0}3 \phantom{0}4 \phantom{0}4 \\ - \phantom{0}0 \phantom{0}1 \phantom{0}8 \phantom{0}7 \\ \hline \phantom{0}2 \phantom{0}1 \phantom{0}5 \phantom{0}7 \end{array}$ <p><math>6467 - 2684</math></p> $\begin{array}{r} \phantom{0}6 \phantom{0}4 \phantom{0}6 \phantom{0}7 \\ \phantom{0}5 \phantom{0}1 \phantom{0}3 \phantom{0}1 \\ \phantom{0}6 \phantom{0}4 \phantom{0}6 \phantom{0}7 \\ - \phantom{0}2 \phantom{0}6 \phantom{0}8 \phantom{0}4 \\ \hline \phantom{0}3 \phantom{0}7 \phantom{0}8 \phantom{0}3 \end{array}$ <p><b>Columnar subtraction (decimals) in contexts such as money and measurement</b> <math>32.34 - 14.18</math></p>	<p><b>Representing problems</b> Check the answer to the following calculations using the inverse. Show all your working.</p>   <p><math>2456 - 734 = 1822</math></p>

2456	
1822	734

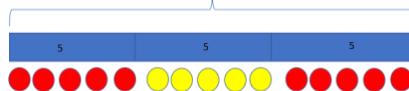
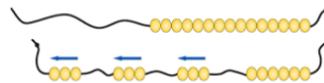
	<p>Place value materials to represent numbers in calculations</p> 	$\begin{array}{r} 754 \quad 600 \quad 140 \quad 14 \\ - 86 \quad \quad \quad 80 \quad \quad 6 \\ \hline 668 \quad 600 \quad 60 \quad 8 \end{array}$ <p>“It’s tricky to take 6 from 4 and 80 from 50. I need to rearrange the number. I will exchange one ten from 50 which leaves 40 and makes 14 in the units. 40 to subtract 80 is tricky. I will exchange one hundred from 700 and make 140. 14 subtract 6 equals 8. 140 subtract 80 equals 60 and 600 subtract 0 equals 600.”</p>	<table border="1" style="width: 100%; text-align: center;"> <tr><td colspan="2">386</td></tr> <tr><td>?</td><td>79</td></tr> </table>	386		?	79	<p>Place value materials to represent calculations Appendix 1.</p>	$\begin{array}{r} 2 \quad 1 \quad 2 \quad 1 \\ ,32,34 \\ -14,18 \\ \hline 18,16 \end{array}$	
386										
?	79									
Known facts	Derive and use addition and subtraction facts to 100, e.g. 33+ 67 =100.		Derive and use addition and subtraction facts (for multiples of 10) to 1000, e.g. 330+ 670=1000.							
Essential knowledge	Subtract single digit bridging through boundaries	Subtract multiples of 10,100	Fluency of 2 digit - 2 digit		Subtract multiples of 10, 100 and 1000					
	Partition second number to subtract	Pairs of 100 (complements of 100)	Partition second number to subtract		Decimal subtraction from 10 or 1					
	Difference between	Subtract near multiples of 10 and 100 by rounding and adjusting	Difference between		Subtract near multiples by rounding and adjusting					
	Partition and recombine									

# MULTIPLICATION KS 1

<p>EYFS</p>	<p><b>Reception: ELG 2018</b></p> <p>Numbers to 20: place them in order and say which number is one more or one less than a given number</p> <p>Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer</p> <p>They solve problems, including <b>doubling, halving and sharing.</b></p> <p><b>Exceeding:</b></p> <p>Estimation and checking quantities by counting up to 20</p> <p><b>Combining groups of 2, 5 or 10 or sharing into equal groups</b></p>			
<p>Year</p>	<p>1</p>	<p>2</p>		
<p>Layers of vocabulary</p>  <p><b>Appendix 1a</b></p> <p>Beck's Tiers of Vocabulary</p> <p><b>Appendix 1b:</b></p> <p>Vocabulary book</p>	<p><b>Basic to subject specific (Beck's Tiers):</b></p> <p>count in ones, twos... tens...</p> <p>array, groups of, equal groups</p> <p>odd, even</p> <p><b>Instructional vocabulary:</b></p> <p>carry on, continue repeat what comes next?</p> <p>find, choose, collect</p> <p>use, make, build</p> <p>tell me, describe, pick out, talk about, explain, show me, read, write, record</p> <p><b>Basic to subject specific (Beck's Tiers):</b></p> <p>lots of, groups of x, times, multiply, multiplied by multiple of once, twice, three times... ten times... times as (big, long, wide... and so on) repeated addition array row, column double, halve share, share equally</p> <p><b>Instructional vocabulary:</b></p> <p>carry on, continue, repeat, what comes next? predict describe the pattern describe the rule</p> <p>find, find all, find different, investigate</p>			
<p>NC 2014</p>	<p>Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.</p> <p>Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (x), division (÷) and equals (=) signs.</p>			
	<p>Concrete, pictorial, abstract</p>			
<p>Developing Conceptual/ Procedural Understanding</p>	<p><b>Grouping</b></p>  <p>2 frogs on each lily pad</p> <p><b>GROUPING ITP</b></p> <p>Pictures to show 2 groups of 3 or 3 groups of 2 etc.</p> <p><b>Doubles</b></p>	<p><b>Arrays</b></p> <p>(rectangular arrangements to show equal groups)</p> 	<p><b>Repeated addition</b></p>  <p>Introduce the x symbol once repeated addition is understood.</p> <p><b>Grouping</b></p>	<p><b>Commutativity</b></p>  <p><math>5 \times 2 = 10</math></p> <p><math>2 \times 5 = 10</math></p> <p><math>5 \times 2 = 10</math></p> <p><math>2 \times 5 = 10</math></p> <p><math>5 \times 2 = 2 \times 5</math></p>



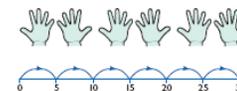
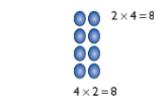
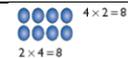
5 frogs on each lily pad  $5 \times 3 = 15$



**Building tables**



Build tables using counting stick- forwards and backwards and with missing jumps



$5 + 5 + 5 + 5 + 5 = 30$

$5 \times 6 = 30$

5 multiplied by 6

6 groups of 5

6 hops of 5

**Decision making**

How many number sentences can you write to describe this array? Can you use addition, multiplication and division?

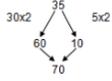
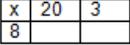
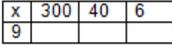
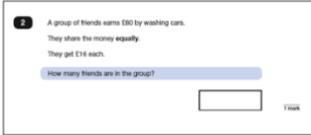


Explain your answers.

Known facts	Count in multiples of twos, fives and tens.		Recall and use $\times$ and $\div$ facts for the 2, 5 and 10 x tables, including recognising odd and even numbers.	
Essential Knowledge	Count in 2s	Doubles up to 10	2 x table	Doubles up to 20
	Count in 10s	Double multiples of 10	10 x table	Doubles of multiples of 5
	Count in 5s	Count in 2s, 5s and 10s	5x table	Count in 3s

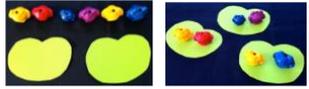
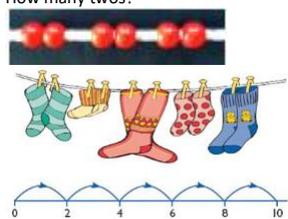
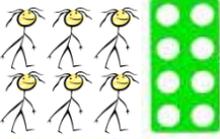
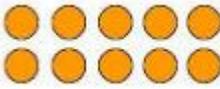
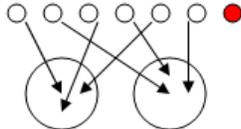
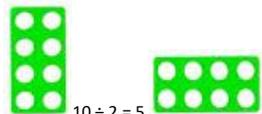
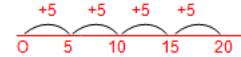
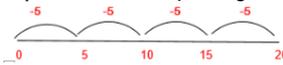
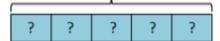
# MULTIPLICATION KS 2

<p>KS1</p>	<p>Pupils should memorise and reason with numbers in 2, 5 and 10 times tables. They should see ways to represent odd and even numbers and know how they are represented in tables. This will help them to understand the pattern in numbers.</p> <p>Pupils should begin to understand multiplication as scaling in terms of double and half (e.g. that tower of cubes is double the height of the other tower).</p> <p>Commutative law shown on array. Repeated addition can be shown mentally on a number line. Inverse relationship between multiplication and division. Use an array to explore how numbers can be organised into groups.</p>	
<p>Year</p>	<p>3</p>	<p>4</p>
<p>Layers of vocabulary</p>  <p><b>Appendix 1a</b> Beck's Tiers of Vocabulary</p> <p><b>Appendix 1b:</b> Vocabulary book</p>	<p><b>Basic to subject specific (Beck's Tiers):</b> lots of, groups of <math>\times</math>, times, multiply, multiplication, multiplied by multiple of, product once, twice, three times... ten times... times as (big, long, wide... and so on) repeated addition array row, column double, halve share, share equally one each, two each, three each...</p> <p><b>Instructional vocabulary:</b> carry on, continue repeat what comes next? predict describe the pattern, describe the rule find, find all, find different, investigate choose, decide, collect</p>	<p><b>Basic to subject specific (Beck's Tiers):</b> lots of, groups of times, multiply, multiplication, multiplied by multiple of, product once, twice, three times... ten times... times as (big, long, wide... and so on) repeated addition array row, column double, halve, factor, multiple</p> <p><b>Instructional vocabulary:</b> carry on, continue, repeat what comes next? predict describe the pattern, describe the rule pattern, puzzle, calculate, calculation, mental calculation, method, jotting, answer right, correct, wrong what could we try next? how did you work it out? number sentence sign, operation, symbol, equation</p>
<p>NC 2014</p>	<p>Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including 2 digit numbers times 1 digit numbers progressing to formal written methods.</p>	<p>Multiply 2 digit and 3 digit numbers by a 1 digit number using formal written layout. Solve problems involving multiplying and adding.</p>

Developing Conceptual/ Procedural Understanding	<b>Building tables</b>  For example, build tables using counting stick- forwards and backwards and with missing jumps <b>Using known facts</b> If $3 \times 2 = 6$ , then $30 \times 2 = 60$ , $60 \div 3 = 20$ and $30 = 60 \div 2$ . <b>Associativity</b> $(2 \times 3) \times 4 = 2 \times (3 \times 4)$ 	<b>Partitioning strategy to double</b> Double 35  <b>Place value materials to represent calculations</b> <b>Partitioning</b> Informal recording of partitioned numbers $15 \times 5 = 75$ $10 \times 5 = 50$ $5 \times 5 = 25$ $27 \times 3 = 81$ $20 \times 3 = 60$ $7 \times 3 = 21$ "20 multiplied by 3 equals 60 and 7 multiplied by 3 equals 21. 60 add 21 equals 81."	<b>Grid method</b> $23 \times 8 =$ $20 \times 8 = 160$ $3 \times 8 = 24$ $23 \times 8 = 184$  <b>Short multiplication</b> Expanded $23$ $\times 8$ $24$ (8 x 3) $160$ (8 x 20) $184$ leading to compact $23$ $\times 8$ $184$ $2$ <b>Representing problems</b> A group of aliens live on Planet Xert. Tinions have three legs, Quinions have four legs. The group has 22 legs altogether. How many Tinions and Quinions might there be? Is there more than one solution?	<b>Building tables</b>  For example, build tables using counting stick- forwards and backwards and with missing jumps <b>Using known facts</b> If $2 \times 3 = 6$ then $200 \times 3 = 600$ and $600 \div 3 = 200$ <b>Distributivity</b> $3 \times (2 + 4) = 3 \times 2 + 3 \times 4$ So the '3' can be 'distributed' across the '2 + 4' into 3 times 2 and 3 times 4  leading to $13 \times 4 = 10 \times 4 + 3 \times 4 = 52$ 	<b>Place value materials to represent calculations</b> <b>Grid method</b> (if needed for conceptual understanding) $346 \times 9$  <b>Short multiplication</b> Expanded $346$ $\times 9$ $54$ (9 x 6) $360$ (9 x 40) $2700$ (9 x 300) $3114$ leading to compact $346$ $\times 9$ $3114$ $45$	<b>Representing problems</b> Multiply a number by itself and then make one factor one more and the other one less. What do you notice? Does this always happen? Eg $4 \times 4 = 16$ $6 \times 6 = 36$ $5 \times 3 = 15$ $7 \times 5 = 35$ Try out more examples to prove your thinking.   Place $<$ , $>$ , or $=$ in these number sentences to make them correct. $50 \times 4$ $4 \times 50$ $4 \times 50$ $40 \times 5$ $200 \times 5$ $3 \times 300$
Known facts	Recall and use $\times$ and $\div$ facts for the 3, 4 and 8 x tables			Recall $\times$ and $\div$ facts for $\times$ tables up to 12 x 12.		
Essential knowledge	Review 2x, 5x and 10x		Double 2 digit numbers		4x and 8x tables	10x bigger
	4x table		3x table		3x, 6x and 12x tables	Double larger numbers and decimals
	8 x table		6x table		3x and 9x tables	11x and 7x tables

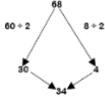
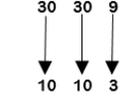
# DIVISION KS 1

<p>EYFS</p>	<p><b>Reception: ELG 2018</b>            Numbers to 20: place them in order and say which number is one more or one less than a given number            Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer            They solve problems, including doubling, <b>halving and sharing</b>.</p> <p><b>Exceeding:</b>            Estimation and checking quantities by counting up to 20  <b>Combining groups of 2, 5 or 10 or sharing into equal groups</b></p>	
<p>Year</p>	<p>1</p>	<p>2</p>
<p>Layers of vocabulary</p>  <p><b>Appendix 1a</b>            Beck's Tiers of Vocabulary  <b>Appendix 1b:</b>            Vocabulary book</p>	<p><b>Basic to subject specific (Beck's Tiers):</b>            count in ones, twos... tens...            share, groups of, equal groups            odd, even</p> <p><b>Instructional vocabulary:</b>            count out, share out, left, left over</p>	<p><b>Basic to subject specific (Beck's Tiers):</b>            share, share equally one each, two each, three each... group in pairs, threes... tens equal groups of <math>\div</math>, divide, divided by, divided into left, left over</p> <p><b>Instructional vocabulary:</b>            tell me, describe, name, pick out, discuss, talk about, explain, explain your method, explain how you got your answer, give an example of... show how you</p>
<p>NC 2014</p>	<p>solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.</p>	<p>Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (<math>\times</math>), division (<math>\div</math>) and equals (<math>=</math>) signs.</p>
	<p>Concrete, pictorial, abstract</p>	<p>Concrete, pictorial, abstract</p>

<p>Developing Conceptual/ Procedural Understanding</p>	<p><b>Grouping/Sharing models</b> Using practical contexts and cross-curricular links (PE) such as socks and shoes; animals in the ark to get into groups. Sharing models such as sharing pieces of fruit.</p> <p>Sharing into equal groups 6 frogs shared equally between 2 lily pads gives 3 frogs on each lily pad or Grouping in equal groups 6 frogs grouped in 2s need 3 lily pads to sit on</p>  <p><b>GROUPING ITP</b> How many twos?</p> 	<p><b>Arrays</b> (rectangular arrangements to show equal groups)</p>    <p><b>Decision making</b> How many cars can you make if you have 8 wheels?</p>  <p>How many different ways can you arrange 12 buttons in equal groups?</p> 	<p><b>Grouping/Sharing models</b> Introduce the ÷ symbol</p>  <p>15 frogs shared equally between three lily pads <math>15 \div 3 = 5</math> or 15 frogs grouped in 5s need 3 lily pads to sit on <math>15 \div 5 = 3</math></p> <p><math>15 \div 3 = 5</math> groups of 3 (grouping)</p>  <p><math>20 \div 2 = 10</math></p>   <p>5 hops in 15. How big is each hop?</p> <p>There are 7 cakes and 2 children. How many cakes will they get each? (Leftovers/reminders introduced)</p>  <p><math>7 \div 2 = 3r1</math></p>	<p><b>Arrays representing the dividend</b></p>  <p><math>10 \div 2 = 5</math> and <math>10 \div 5 = 2</math></p> <p><b>Repeated addition (to reach a given target)</b></p>  <p>There are 20 sweets in a bag. How many children can have 5 each?</p>  <p><b>Repeated subtraction (from a given quantity)</b></p>  <p><b>Links to tables</b></p>  <p>Use language of division linked to tables using counting stick</p> <p><b>Representing problems</b> Jane has 30 cakes. She wants to share them equally between 5 boxes. How many cakes should go in each box?</p>  <p>Number of cakes in each box = 6 <math>30 \div 5 = 6</math></p>
<p>Known facts</p>	<p>Count in multiples of twos, fives and tens.</p>		<p>Recall and use <math>\times</math> and <math>\div</math> facts for the 2, 5 and 10 <math>\times</math> tables, including recognising odd and even numbers.</p>	
<p>Essential Knowledge</p>	<p>Count back in 2s</p>	<p>Halves up to 10</p>	<p>Division facts (2 <math>\times</math> table)</p>	<p>Halves up to 20</p>
<p></p>	<p>Count back in 10s</p>	<p>Halve multiples of 10</p>	<p>Division facts (10 <math>\times</math> table)</p>	<p>Review division facts (2 <math>\times</math>, 5 <math>\times</math>, 10 <math>\times</math> tables)</p>
<p></p>	<p>Count back in 5s</p>	<p>How many 2s? 5s? 10s?</p>	<p>Division facts (5 <math>\times</math> table)</p>	<p>Count back in 3s</p>
<p>Tests of divisibility</p>	<p>All even numbers will divide by 2</p>		<p>All numbers ending in 0 will divide by 10</p>	<p>All numbers ending in 5 and 0 will divide by 5</p>

## DIVISION KS 2

<p>KS1</p>	<p>Noticing how counting in multiples of 2, 5 and 10 relates to the number of groups you have counted (introducing times tables) links to division.</p> <p>An understanding of the more you share between, the less each person will get (e.g. would you prefer to share these grapes between 2 people or 3 people? Why?)</p> <p>Secure understanding of grouping means you count the number of groups you have made. Whereas sharing means you count the number of objects in each group.</p>	
<p>Year</p>	<p>3</p>	<p>4</p>
<p>Layers of vocabulary</p>  <p><b>Appendix 1a</b> Beck's Tiers of Vocabulary</p> <p><b>Appendix 1b:</b> Vocabulary book</p>	<p><b>Basic to subject specific (Beck's Tiers):</b> share, share equally one each, two each, three each... group in pairs, threes... tens equal groups of <math>\div</math>, divide, division, divided by, divided into left, left over, remainder, dividend, divisor</p> <p><b>Instructional vocabulary:</b> calculate, work out, solve, investigate question, answer, check</p>	<p><b>Basic to subject specific (Beck's Tiers):</b> share, share equally one each, two each, three each... group in pairs, threes... tens equal groups of <math>\div</math>, divide, division, divided by, divided into left, left over, remainder, dividend, divisor</p> <p><b>Instructional vocabulary:</b> calculate, work out, solve, investigate, question, answer, check</p>
<p>NC 2014</p>	<p>Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including 2 digit numbers times 1 digit numbers progressing to formal written methods.</p>	<p>Practise to become fluent in the formal written method of short division with exact answers.</p>

Developing Conceptual/ Procedural Understanding	<b>Links to tables</b>  For example, use language of division linked to tables using counting stick <b>Using known facts</b> If $3 \times 2 = 6$ , then $30 \times 2 = 60$ , $60 \div 3 = 20$ and $30 = 60 \div 2$ . <b>Partitioning strategy to halve</b> Halve 68  <b>Rearranging the dividend to find multiples of the divisor.</b> $48 \div 3 =$ 'What do I know about the 3 x tables?' "I know $3 \times 10 = 30$ and $3 \times 6 = 18$ ."  $48 \div 3 = 16$		<b>Place value materials to represent calculations</b> <b>Short division</b> $72 \div 3 =$ $3 \overline{) 72}$ $\begin{array}{r} 24 \\ 3 \overline{) 72} \\ \underline{6} \phantom{0} \\ 12 \\ \underline{12} \\ 0 \end{array}$ '72 divided by 3. 7 tens shared equally between 3 is 2 with a remainder of 1 ten. Exchange the 1 ten for 10 units. I now have 12 units which shared equally between 3 is 4. The answer is 24.' <b>Representing problems</b> Andy says 'I can use my three times table to work out $180 \div 3$ '. Explain what Andy could do to work out this calculation.		<b>Links to tables</b>  For example, use language of division linked to tables using counting stick <b>Using known facts</b> If $2 \times 3 = 6$ then $200 \times 3 = 600$ and $600 \div 3 = 200$ <b>Rearranging the dividend to find multiples of the divisor.</b> $69 \div 3 =$ 'What do I know about the 3 x tables?' "I know $3 \times 10 = 30$ and $3 \times 3 = 9$ ."  $69 \div 3 = 23$ $3 \overline{) 241} \\ \underline{6} \phantom{0} \\ 13 \\ \underline{12} \\ 11 \\ \underline{9} \\ 21 \\ \underline{21} \\ 0$		<b>Place value materials to represent calculations</b> <b>Short division</b> $372 \div 6 =$ $6 \overline{) 372}$ $\begin{array}{r} 62 \\ 6 \overline{) 372} \\ \underline{36} \phantom{0} \\ 12 \\ \underline{12} \\ 0 \end{array}$ '372 divided by 6. 3 hundreds cannot be shared equally between 6, so exchange the hundreds for 30 tens. I now have 37 tens which shared equally between 6 is 6 with a remainder of 1 ten. Exchange the ten for 10 units. I now have 12 units which shared equally between 6 is 2. The answer is 62.' <b>Representing problems</b> Alan says that the solution to $186 \div 4$ can be written as '46 remainder 2' or as '46.5'. Do you agree? Explain your answer.	
	Known facts	Recall and use $\times$ and $\div$ facts for the 3, 4 and 8 x tables		Recall $\times$ and $\div$ facts for $\times$ tables up to $12 \times 12$ .				
Essential knowledge	Review division facts (2 x, 5 x and 10 x tables)	Halve 2 digit numbers	Division facts (4x and 8x tables)	10x smaller				
	Division facts (4 x table)	Division facts (3 x table)	Division facts (3 x, 6 x and 12 x tables)	Halve larger numbers and decimals				
	Division facts (8 x table)	Division facts (6 x table)	Division facts (3 x and 9 x tables)	Division facts (11 x and 7 x tables)				
Tests of divisibility	KS1: 2, 5, 10	Any number with a digit sum of a multiple of 3, will divide equally by 3	Any number with a digit sum of a multiple of 3, will divide equally by 3 KS1: 2, 5, 10	Any number with a digit sum of a multiple of 3 and is even will divide equally by 6				